# How To Distribute The Difficulty of Problems On A Test

By: Guodong Zhang from Team #864

# Summary

Examination has always been used by professors to appraise the students regarding the quality of education. Specifically, a group of well-organized testing problems is able to reflect the amount learned and understood by each individual, as well as the condition of the whole class. Therefore, there is no doubt that the professors, when choosing testing problems, should consider the difficulty of each problem. We may create a model and utilize corresponding and relevant mathematical calculation to give some helpful advice regarding how to distribute the level of problems.

As a 10th grader in high school, my background knowledge about modeling is not enough, or should I say, far from enough to come up with a perfect way of solving this problem. However, I do realize that modeling plays an increasingly significant role in various subjects and in our daily life as well. Therefore, I would like to participate in this sort of competition and challenge myself while learning more. What's more, I am looking forward to receive professional replication and suggestion from the judges so that I can do better next time. I will try my best to write this paper. My teammates, Xuanang Xiong and Hongkai Yu will figure something out about how to launch GPS effectively in order to make it helpful to most of the people.

Speaking of "ideal distribution", there is no definite way to define what kind of distribution is ideal. However, considering the purpose of exam and it is easy to see that the level on which the ability of each individual can be reflected accurately is the number one choice.

The difficulty of a problem can be defined by the number of students who solve it successfully. By comparing the ability and the difficulty of the problem, the capability for an individual to solving the problem can be determined to be either "can" or "cannot" . And by graphing the result, we can directly view the distribution.
After reasoning, calculating and graphing, I came to the conclusion that the difficulty levels of the problems should be equally divided and distributed from the easiest to the hardest, thus directly and accurately reflecting the corresponding studying status of each individual student.

I would like to say thank you to everyone from AoCMM for providing me this opportunity to take part and learn, and to the judges for your work and helpful advise. I appreciate that.

# Introduction

## The original problem

How should a professor distribute the difficulty of problems on a test to ensure that a group of student, with varying abilities, will form an ideal distribution?

# Assumptions

## 1.The difficulty level of problem (DLOP) can be represented numerically.

In the range from 1 to 100, each integer represents one difficulty level of a problem appeared on the test paper. Level 1 is the easiest and 100 the hardest.

## 2.The ability to solve problem (ATSP) of each student can be represented numerically.

Based on the intrinsic talent and potential, the quality of pre-study, the attention paid to this specific class and to every lecture as well, the attitude of finishing the daily task, the proactivity of communicating with professor and so on, the ability for an individual student to solve problem on the test is determined and can be represented numerically. In the range from 1 to 100, each integer represents one level of the ability to solve problem. Level 1 is the lowest and 100 the highest.

## 3.Whether or not a problem can be solved by a student is determined by comparing the DLOP of the problem to the ATSP of the student.

If DLOP is larger than or equal to ATSP, the problem can be solved by the student. Otherwise, it can not be solved.

## 4.No emergency that affect either the DLOP or the ATSP would take place during the test.

# Model

1.If the professor distribute the DLOP equally from 1 to 100, then when a student takes the test, he can solve problems whose DLOP is lower than or equal to ATSP. However, when confronting the problem whose DLOP is higher than ATSP of the student, he fails to answer it correctly. When every student goes through the process, the number of problems which are solved successfully accurately and directly reflect the ability of every individual. The result will distribute the same way as the various abilities of students distribute, thus can be considered "ideal" since the purpose of examination is to appraise the quality of studying.

Specifically, suppose there are 100 problems whose DLOP is from 1 to 100 respectively, and there are 100 students as well whose ATSP distribute equally in the range of 1 to 100. See figure 1.1 to see the distribution of DLOP, figure 1.2 the ATSP, and figure 1.3 the scores.

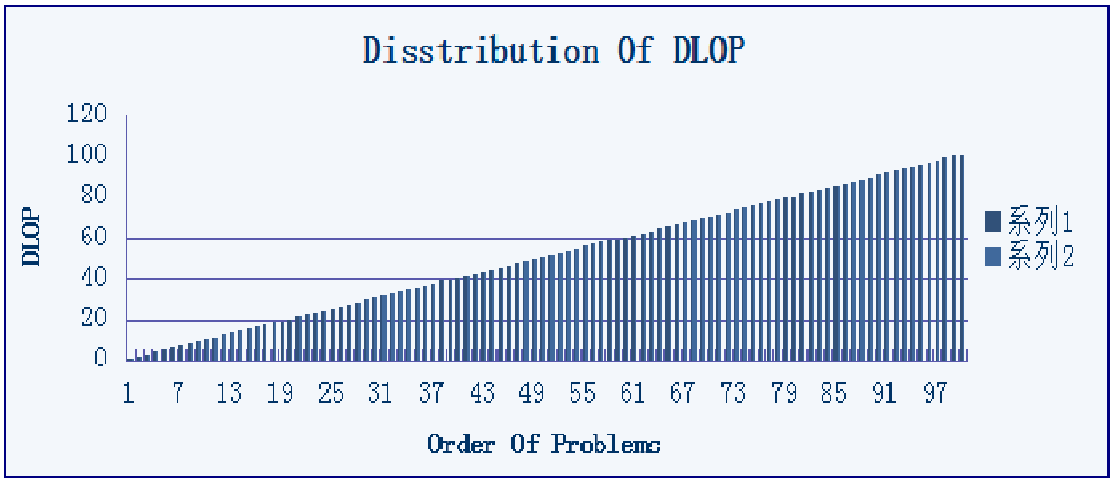


Figure 1.1

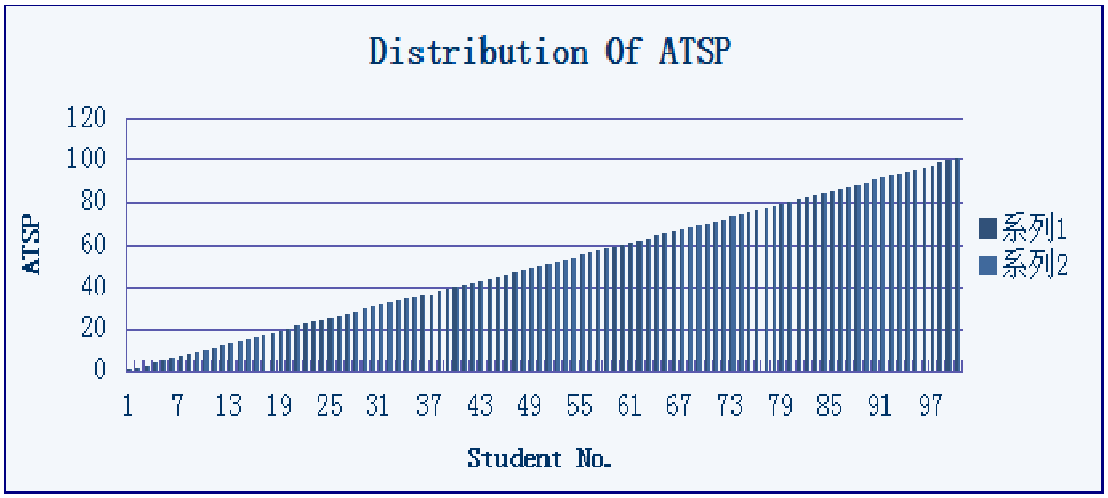


Figure 1.2

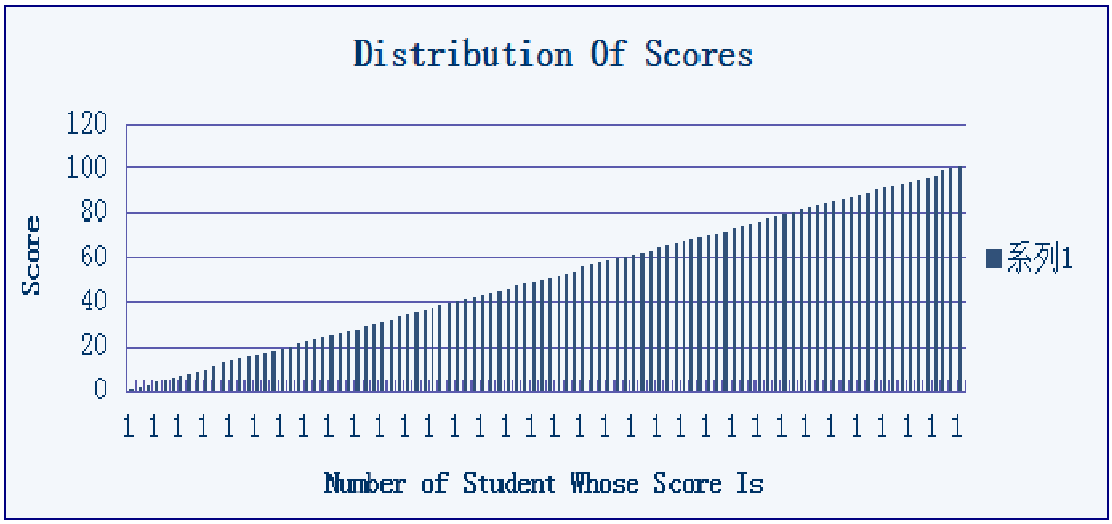


Figure 1.3

# Conclusion

It is obvious from the figure that the three distribution is exactly of the same shape, indicating that equally distribute the DLOP in test is an effective way to test the level below which the ATSP of a student is greater than the DLOP. Therefore, the score directly and accurately reflects the level of the individual student.

# Further Inference

Generally considering, if the professor is intended to know how well and how deep did the class understand the knowledge and is able to utilize, he can take a look at the average score or median score of the class, which reflects the general level of the whole class. The higher, the better.

# Sensitivity Analysis

As a 10$^{th}$ grade high school student, this is the best way I can come up with currently to solve the problem. The strengths are as clear and obvious as the weaknesses are.

## Strengths

1.It is my original idea rather than derivative ones or those come up under the help of others. I hate plagiarism and dishonest under any circumstances.

2.It provides a theoretically perfect model of organizing test problems, by which the result of test is correspond to the ability of contestant directly and accurately.

## Weaknesses

1.It does include neither advanced mathematical formulas nor the application of computer programs. Therefore, it can not be as comprehensive and convincing as those which are designed by undergraduates who get more acquaintance with relevant background knowledge and who are more experienced in writing papers.

2.When it comes to authentic situations, it is hard to operate and carry out such a theoretical conclusion. For example, it is hard to specifically evaluate the difficulty level of each problem, nor it can be ensured that something irrelevant may happen and thus influence the result of the rest. If a student is suffering from headache, there is a strong likelihood that the quality of his answer can be degraded.

# Citations

1.*University of Tongji, Collection and Interpretation of Advanced Papers in American Mathematics Modeling Competition*, written by Modeling Committee of University of Tongji, published by TONGGJI UNIVERSITY PRESS, 2014.11.
2.*Application of Modeling*, written by Qiyuan Jiang and Jinxing Xie, published by Advanced Education Press, 2014.8.

# Appendix

## Notations

| | |
|---|---|
| DLOP | The difficulty level of problem |
| ATSP | The ability to solve problem |

## Applications

Microsoft Office Word(Windows 2007)
Microsoft Office Excel(Windows 2007)
Foxit Reader

# GPS In The Future

By: Xuanang Xiong, Hongkai Yu from team #864

# Summary

We are happy to model in the competition. Sorry referee for our English is not good like group mate Guodong Zhang. I hope you can understand us.
After we read the words online, we think that we never learned something about GPS. It is too quickly to more informations. But, we using our physics knowledge and can make a understanding model.

# Introduction

We would like to position N GPS satellites, making sure that at least 90% of the world's population can get their location using the GPS system accurately at least 95% of the time. What is N and what is the constellation needed to achieve it?

# Assumptions

First, the question told us that "a minimum of 4 satellites are needed to get a solution". So, N is 4.
Second, we think in physics we need two points as reference to make sure our location of place. Let's see how to make the 4 satellites circle around the Earth so that on every position we can connect to 2 of them.

# Model

Cut the Earth from the equator into the northern one and the southern one. Cut it from any line circle that is 90 degree with the equator, and the Earth is separated into the left one and the right one.
Let two of the satellites be Group 1 and go around the Earth on the equator but opposite to each other if the center is the center of the Earth. They should be far from the planet so that everyone one the side can connect to them.
The other two belong with Group 2 and should go in the opposite direction to the 2 before. Same to the 2 above equator, these two should be far enough so that everyone can connect one of them.
From anywhere in the world, GPS can be connected to one satellite in Group 1 and one in Group 2. This can make 100% working a reality.

#864

# Strengths and Weaknesses

1.Maybe because we have not gone to university yet, we do not know better answers.
2.Sorry for our English. We do not want our English teacher or GuodongZhang to help because practicing makes perfect. This is a good chance to practicing our English. Thank you very much!
3.Beside those bad points, this can be a workable way to do what is asked in the question. Maybe it would someday comes true!